PRICING BEHAVIOR OF MULTI-PRODUCT RETAILERS

Daniel Hosken and David Reiffen

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ABSTRACT

Two common features of retailing are that each retailer sells many different products, and that pricing strategies differ across these products. This paper extends previous theoretical research on single-product retailer competition to a multi-product setting. Specifically, we model a retailer's optimal pricing strategy for perishable and non-perishable items. We find the intuition used to explain retailer behavior in single-product models generalizes to the multi-product setting. Moreover, the multi-product setting allows us to generate a richer set of implications than does the single product case, some of which we empirically examine. Consistent with the theory, price changes are significantly less frequent for the non-perishable item examined (peanut butter) than for the perishable good (margarine), but when price changes occur, they are larger in magnitude for the non-perishable. Further, perishable and non-perishable price changes are negatively correlated, as the theory predicts. We view this evidence as suggesting that retailers' pricing strategies are related in predictable ways to product characteristics, such as consumers' storage costs.

I. Introduction

Empirical and theoretical treatments of retailing (and distribution more generally) have been relatively scarce in the economics literature. At the same time, government enforcement actions are increasingly directed toward such industries.[1] Understanding what retailers do, and how they compete seems essential to understanding the competitive impact of mergers.

Casual empiricism suggests that retailers use different selling strategies across the set of goods that they carry. In particular, the frequency and magnitude of *sales* differ across the set of goods that most retailers sell.[2] Several previous papers (for example, Sobel (1984), Pessendorfer (1997), and Varian (1980)) have sought to explain the frequency and magnitude of *sales* for a single-product retailer. The primary goal of this paper is to generalize these single-product models to an environment in which each retailer sells multiple products. Specifically, we determine equilibrium pricing behavior over time in a model in which competing retailers sell two goods, one of which can be inventoried by consumers (non-perishables), one of which cannot (perishables). We show that the basic implications of the single-product models carry through to the multi-product case; for example, price reductions occur periodically without any change in costs. However, our model also generates additional implications for the relationship between the prices for the two kinds of goods. For example, we show that price movements for the two goods should be negatively correlated, and that price discounts for non-perishables goods should be deeper, but less frequent.

We test the predictions of our model using publicly-available store-level scanner data on prices from supermarkets in two Midwestern cities. One test is a comparison of the frequency and magnitude of sales for one non-perishable product (peanut butter) and one perishable product (stick

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[1] Recent examples of government actions against distributors include litigations to stop mergers in the office supply retailing (*FTC v. Staples Inc., D.C.C.*, 7/30/97), and drug wholesaling (*FTC v. Cardinal Health, Inc*., and *FTC v. McKesson Corp., D.D.C.,* 7/31/98) industries.

[2] We use the term *sale* to refer to a temporary reduction in the price of an item which is unrelated to cost changes. The price reduction is temporary in that consumers know that the retailer will raise his price in the near future.

margarine) over a two-year period.[3] Consistent with the model, we find that price discounts for peanut butter are less frequent but of larger magnitude than discounts for stick margarine. Further, perishable and non-perishable price changes are negatively correlated, as the theory predicts.

We view this evidence as suggesting that price discrimination through intertemporal price changes is one function served by sales in the food retailing business. To the extent these findings are confirmed in future research, they would have several implications for interpreting empirical results. For example, they would suggest that for certain types of goods, the elasticities derived from estimating demand using contemporaneous price and quantity data will not answer the question of how consumption would change if the entire distribution of retail prices changed (e.g., because of a change in wholesale price). Elasticities derived in this manner can be thought of as *purchasing* elasticities, measuring the response of consumer buying behavior to temporary changes in price. This can be quite different from *consumption* elasticities (which measure the response of purchases to permanent price changes) if purchasing behavior has an intertemporal component (due to, e.g., consumer inventorying). To a manufacturer contemplating a change in its wholesale price, or an antitrust agency evaluating the effect of a merger of two manufacturers, it is the consumption elasticity that is relevant to assessing the impact of a price change.

II. Previous Explanations of Sales

Sales, in the sense of periodic, temporary reductions in specific product prices are a feature of supermarket competition, but one which has not generated a great deal of economic research. We primarily draw on the model first developed by Conlisk, Gerstner and Sobel (1984) to analyze this behavior. The basic intuition in their model is that consumers differ in demand elasticity and in their willingness to wait (which is analytically similar to differences in costs of inventorying). If these differences are correlated (low elasticity customers are also less willing to wait), a seller can price discriminate by making high-elasticity customers wait for low prices. Hence, *sales* arise because these periodic price reductions lead to a large volume of purchases by high-elasticity

[3] Peanut butter fits our notion of non-perishable very well, while stick margarine is somewhat further from our ideal perishable, but was the closest among the products for which data were available.

customers, while allowing the seller to charge high prices most of the time to low-elasticity customers.

The Conlisk et al. model captures this intuition in a tractable environment. In their model, there are two classes of consumers: One class has a high reservation value to consuming the good (a_H) and an infinite discount rate, and a second class has a lower value (a_L) and a finite discount rate. The seller cannot determine an individual consumer's type and hence must charge the same price to everyone in each period. One cohort of each type of consumer enters the market in each period, and then each consumer departs the market as soon as she purchases one unit of the good. That is, consumers still in the market with a reservation value below that period's market price do not purchase during that period but remain willing to buy (at a sufficiently low price) in future periods.

Conlisk, et al. show that given these consumer preferences, a monopoly retailer of the good would charge a_H for a number of periods, thereby capturing the value the high-value consumers place on the product. During any period in which the retailer charges a_H, low-value consumers do not make a purchase but remain willing to buy in the future if price declines sufficiently. Eventually, as the number of unsatisfied low-value consumers grows, it becomes profitable to lower prices sufficiently to sell to the large group of low-value consumers that have "accumulated." By having a sale, the retailer's profits from selling to high-value consumers falls (by a_H-a_L), but is made up for through the high volume of sales to low-value consumers. Thus, Conlisk, et al. provide an explanation of periodic sales; whereby a retailer lowers its price for a short time, even though its costs and the number of new high-value consumers has not changed.[4] In this model, sales can be seen as a means of price discriminating against impatient, high-value consumers.[5] Sobel (1984) extends this model to the case of multiple retailers. Sobel interprets the high-value

[4] One can interpret the Conlisk, et al. result as providing an economic explanation for the famous retailing cliche "to make it up in volume."

[5] The same general model has been used to explain the use of targeted price cuts (e.g., manufacturer coupons). In Banks and Moorthy (1999), high and low value consumer differ in their costs of obtaining the price discount (e.g., the cost of using coupons), rather than in their willingness to wait/inventory. Given the differential search cost, a coupon is a means of offering low prices to low-value/ low search cost consumers while simultaneously charging high prices to high-value consumers.

consumers as not only willing to pay more for the good, but also as being loyal to one retailer. Thus, with J identical retailers, by charging a_H each retailer can earn revenues of (a_H/J) times the number of high-value customers. In addition to being more willing to wait than high-value consumers (as in Conlisk et al.), low-value consumers are willing to buy at whichever retailer offers the lowest price. The primary difference between this model and the monopoly model is that low-value consumers do not accumulate in the same way when a retailer charges a high price. Here, the low-value consumers accumulate in aggregate, but they react to a lower price charged by any retailer. Hence, an individual retailer may miss the opportunity to sell to the accumulated low-value consumers after charging a_H for a number of periods because the low-value consumers may have purchased elsewhere. In the multiple retailer model, each retailer faces the same basic decision; is it preferable to sell to the group of loyal customers at a high price, or cut price and sell to both these customers and the accumulated non-loyal consumers before a rival does? As the length of time since any retailer had a sale increases, the number of non-loyal consumers rises as well, and this later option becomes more attractive. In the equilibrium in Sobel's model, all retailers charge a_H for a number of periods, until the expected profit from selling to the accumulated low-value consumers at a low price equals the profit from selling to the loyal customers at a high price. At this point, each retailer chooses a price from a continuous distribution of prices.

The basic characteristics of the equilibrium in Sobel resembles the monopoly case; retailers charge a_H when the number of non-loyal customers is small, but as the number grows, it eventually becomes profitable to reduce price in order to attract the non-loyal customers. The key difference between the monopoly and multiple retailer equilibria is that sales occur more frequently (and at lower prices) with multiple retailers. Finally, one can extend the model to show that the difference between the monopoly and multiple retailer cases is a general one. That is, a reduction in the number of competing retailers has the effect of reducing the frequency and depth of sales.

Pesendorfer (1997) both simplifies and generalizes the Sobel model. The simplification is that he assumes low-value customers do not behave strategically - which is to say that they buy whenever price is below a_L.[6] The generalization is that Pesendorfer allows some portion of low-

[6] In contrast, in Sobel's model, low-value consumers may wait to buy, even if price is

(continued...)

value consumers to be store-loyal. The Pesendorfer model is formally equivalent to a model in which both types of consumers consume one unit of the good in every period (rather than exit the market as soon they purchase one unit), but the low-value consumers consume from their own inventory whenever the price is above a_L.[7]

While this model explains price discounts for goods that can be inventoried, or goods that are infrequently purchased, it does not explain discounts for perishable goods that are frequently purchased and not inventoried, such as dairy products and produce. However, the evidence (see Section IV) suggests that prices of these items also vary considerably over time. Varian (1980) provides a related explanation for periodic sales of these products. As in the model described above, Varian assumes that some customers are store loyal (buying from their preferred retailer as long as that retailer's price is below the consumer's reservation price), and others buy from the store with the lowest price. Retailers then choose between obtaining a high price, and selling only to store-loyal customers, or charging a "low" price and potentially selling to non-loyals as well. Varian shows that the only symmetric equilibrium features mixed strategies, where all retailers choose their price from a continuous distribution.

Note that the reason for sales in the Varian model is quite different from the reason in Conlisk et al. In Varian, sales result from competition between imperfectly-competing retailers; a monopoly retailer would not vary price. In contrast, the Conlisk et al. model is a monopoly model, and sales are a means of price discrimination. Sales in the Sobel and Pesendorfer models combine elements of both explanations.

One interesting contrast between the Varian model of sales on the one hand and the Conlisk et al., Sobel and Pesendorfer models on the other is that the kinds of price changes predicted for

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[6] (...continued)
below a_L, if they expect price to fall further. Sobel shows that the expected price decline eventually dissipates, and that consumers rationally purchase the good. Thus, the qualitative predictions of the Pesendorfer version are similar to Sobel's results.

[7] This formal equivalence require that low-value consumers have some inventory at the beginning of period 1, and that when price is below a_L, these consumers buy a sufficient quantity for storage to replace the inventory consumed since the previous sale. These assumptions are discussed further in Section III.

goods which can be readily inventoried differ from those for perishable goods. The three models of price discrimination through time-variant pricing suggest that the former type of good will be characterized by a long period of constant prices followed by a large price reduction for a short period. In contrast, the Varian model predicts that price changes for perishable goods will be smaller and more frequent.

In the context of this paper, we use the term *sales* to refer to periodic temporary reductions in prices that are unrelated to cost changes. Several other kinds of systematic price reductions have been documented. One pattern is that prices for goods with a "fashion" element often systematically decline over a fashion season (see, e.g., Pashigian (1988), Pashigian and Bowen(1991)). Warner and Barsky (1995) provide additional evidence of this pattern for fashion goods. They study the pricing of 7 durable goods, and find that prices for the one good that has a fashion element (sweaters) seems to decline systematically over the season, while there is no such pattern for the other 6 goods. Another regularity they find that applies to all 7 goods is that a disproportionate number of price declines occur on Fridays, which are often reversed the following Monday.[8] While our analysis does not directly address either of these regularities, Warner and Barsky's explanation of this week-end effect involves price discrimination by inter-temporal price changes, similar to that advanced by Conlisk et al., Sobel and Pesendorfer.

III. Sales and Multi-Product Retailers

The models described in the previous section all dealt with how a single-product retailer would adjust his prices over time. The phenomenon these models seek to explain is the pattern of price changes that seem to characterize retailer behavior in some industries. Of course, actual retailers carry a large number of individual products. In evaluating whether these models explain retailer pricing behavior, it is important to consider whether these results also holds in a multi-product environment. In this section, we analyze competition between retailers, each of whom sells two kinds of products. We show that in the simple multi-product environment, the same basic forces that result in sales in single-product models do generalize to the multi-product environment.

[8] As discussed in Section IV, they also find the periodic and significant price reductions predicted for the kinds of infrequently-purchased goods they study.

In addition, modeling retailers as multi-product sellers generates additional implications for prices. For example, we show that prices for the two goods should vary inversely.

An important aspect of the nature of multi-product retailers is that most consumers buy an array of goods each time they visit certain kinds of retailers (especially supermarkets). Our model incorporates this feature by assuming that consumers know all of the relevant prices before visiting any outlet, and shop at no more than one store in each period. This means that if a consumer purchases both goods in the same period, she buys both of them from the same retailer.[9] It follows that retailers compete for customers by attempting to offer the most attractive bundle of prices.

In analyzing what constitutes the most attractive bundle, it is necessary to consider the number of units of a good that a consumer may purchase during each visit. Of particular interest to us is whether a consumer can economically buy more units of a good than she plans to consume in that period, inventorying a portion for later consumption. To the end, we designate one of the two goods as a non-perishable, and the other as a perishable. The key difference between perishable and non-perishable goods in our model is that, at some cost, non-perishable goods can be inventoried by the consumer, whereas perishable goods have to be purchased each period. All stores sell the same assortment of non-perishable goods and perishable goods. We refer to the non-perishable good as N, and the perishable as P.

In both the Sobel/Pesendorfer and Varian models, there are two types of consumers; those that are store-loyal and those that shop across stores for the lowest price. In both sets of models, all consumers have a unit demand for consuming each good in each period as long as the price of the good is below their reservation value for the good, and the seller cannot determine an individual consumer's type. In the Sobel model, store-loyals have a sufficiently high cost of waiting (or equivalently, of storing goods) such that only non-loyals choose to wait for lower prices. To make

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[9] This assumption can be explicitly traced to the underlying transactions cost of visiting multiple outlets, but adding this additional notation adds little to the analysis and is omitted. For example, one could interpret the reservation price on the perishable as being defined net of the transactions cost of visiting one supermarket, and that the transactions cost of visiting a second retailer exceeds this reservation value. Salop and Stiglitz (1982) make a similar assumption about the cost of visiting multiple retailers in their analysis of sales. Warner and Barsky (1995) also rely on the transactions cost of visiting multiple retailers to explain the empirical regularity of lower prices on weekends.

this distinction as clear as possible, we assume that store-loyals have infinite storage costs, and non-loyals have zero storage costs (any significant difference in customer's storage cost is sufficient for our purposes). Sobel also assumes that the value store-loyal customers place on the good (a_H) is higher than the value that non-loyals (a_L) place on it. In contrast, in Varian, all consumers have a reservation value of ß for the good. We combine these assumptions, by allowing non-loyals to have a reservation value of a_L for the non-perishable and ß for the perishable. In contrast, store-loyal customers have a reservation value of a_H to buying the non-perishable at their preferred store, and ß to buying the perishable at their preferred store. This implies the following about consumer behavior: Letting P_P be the price of the perishable and P_N the price of the non-perishable at her preferred store, a store-loyal customer will make one of four choices in any period;

if $P_N > a_N$ and $P_P > ß$	buy nothing
if $P_N \leq a_N$ and $P_P > ß$	buy one unit of the non-perishable only
if $P_N > a_N$ and $P_P \leq ß$	buy one unit of the perishable only
if $P_N \leq a_N$ and $P_P \leq ß$	buy one unit of each good

The non-loyal customers also make one of 4 choices. Suppose there are J retailers and let the superscript j index the specific store, then a non-loyal customer's choices are

if $\min_j (P_N^j) > a_L$ and $\min_j (P_P^j) > ß$	buy nothing
if $\min_j (P_N^j) > a_L$ and $\min_j (P_P^j) \leq ß$	buy one unit of the perishable at lowest-price store
if $\min_j (P_N^j) \leq a_L$ and $\min_j (P_P^j) > ß$	buy multiple units of the non-perishable at the lowest-priced store
if $\min_j (P_N^j) \leq a_L$ and $\min_j (P_P^j) \leq ß$	buy the perishable or multiple units of the non-perishable (or both) at whatever store offers the greatest consumer surplus.

The difference between the fourth option in the two cases illustrates an important component of shopping in our model. A store-loyal consumer's decision rule regarding her purchases of the two products are independent; she buys one unit of good i at her preferred store if good i's price is below her reservation value for good i, without reference to good k's price. In contrast, the purchasing decisions for the two goods are linked for non-store loyal consumers. Since by assumption consumers visit at most one store per period, these consumers must determine the consumer surplus offered by each store and choose the store that offers the largest consumer surplus.[10] Depending on the prices of the perishable and non-perishable items, they may buy the perishable, multiple units of the non-perishable, or both goods.

As long as $P_P^j \leq ß$ and $P_N^j \leq a_H$, customers loyal to retailer j will buy both products at that store. Indeed, if retailers only cared about selling to store-loyals, they would always charge $P_P = ß$ and $P_N = a_H$.[11] The reason that retailers might offer lower prices is that non-loyals choose between retailers on the basis of the consumer surplus they can obtain. A non-store-loyal's consumer surplus from buying the perishable is $a_L - P_N$ times the number of units purchased. To conform with the models described in the previous section, we assume these consumers buy a sufficient quantity of the non-perishable to replace the amount they consumed since the previous sale.[12] Let d_j be the consumer surplus retailer j offers non-store-loyal customers and M be the number of periods since the last sale. Formally, $d_j = \{\max[0,(M+1)(a_L - P_N^j)] + \max[0, ß - P_P^j]\}$. Whether retailer j makes any sales to non-store loyals depends on how d_j compares to the d_i offered by each other retailer i.

To reduce notational complexity, we interpret a_L, a_H and ß as the difference between the consumer's reservation value and the constant marginal cost of selling the good, so that we

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[10] In contrast to the model in Lal and Matutes (1994), we assume that consumers are fully informed about the prices charged.

[11] If $P_P > ß$ (or $P_N > a_H$), then retailer j makes no sales of the perishable (non-perishable). Hence, we restrict the analysis to values of $P_N \leq a_H$ and $P_P \leq ß$.

[12] Following Pesendorfer, we assume that the decision rule of low-value consumers is to buy the non-perishable whenever $P_N < a_L$. Clearly, the assumption that consumers exactly replace their depleted inventory is not derived from a model of optimal consumer inventory behavior. This omission is not critical in that the only property of inventory behavior that is required for our results is that when a *sale* occurs, aggregate purchases of the good by low-value consumers is increasing in the length of time since the previous *sale*. This property holds for some simple inventory models that we investigated. For this reason, our model does not require identical inventorying behavior by all low-value consumers.

normalize the retailers' cost to zero. Additionally, we normalize the number of customers to one. Given these assumptions, the implications for retailer profits of these specifications about consumer behavior are as follows. Suppose that a portion, ? (where ? < 1) of customers are store-loyal, and (1-?) are non-loyals. Retailers are assumed to be symmetric, so that ?/J are loyal to each store. One strategy for retailer j is to charge $P_N^j = a_H$ and P_P^j = ß, which results in $d_j = 0$ for all non-loyals. This yields profits of ?(a_H + ß)/J + (1-?) ß/J if all rival retailers choose these same prices, and ?(a_H + ß)/J if any rival retailer chooses to offer $d > 0$. An alternative strategy is to have a sale on the perishable only, so that P_P^j < ß and $P_N^j = a_H$. This yields profits of ?($a_H + P_P^j$)/J + (1-?) P_P^j if retailer j offers the highest d. Finally, he can have only the non-perishable on sale so that $P_N^j < a_L$ and P_P^j = ß, and retailer j will earn profits of ?(ß + P_N^j)/J + (1-?) [(M+1)P_N^j + ß)] if $d^j < \max_{i...j} (d^i)$ and ? (P_N^j +ß)/J otherwise.

We now proceed to derive equilibrium pricing for the two goods, under the assumption that retailers are all risk neutral. Our first result is that retailer j will at most, put one good on sale.

Proposition 1: Retailer j will never choose to put both goods on sale simultaneously (i.e., will never charge $P_N < a_H$ and $P_P <$ ß).

Proof: Charging $P_N < a_H$ and/or $P_P <$ ß is profitable to retailer j iff it results in non-loyals obtaining higher d by buying from retailer j than buying from any other retailer. Retailer j's profit is

$$\frac{?}{J}(P_P\%P_N)\%(1\&?)Prob(\mathrm{d}_j{>}\mathrm{d}_{\&j})(P_P\%(M\%1)P_N) \quad if \ P_N{<}\mathrm{a}_L$$
$$\frac{?}{J}(P_P\%P_N)\%(1\&?)Prob(\mathrm{d}_j{>}\mathrm{d}_{\&j})P_P \qquad\qquad if \ P_N{>}\mathrm{a}_L$$

where $d_{-j} = \max_{i \dots j} (d_i)$. If retailer j chooses P_N and P_P such that $a_H > P_N \geq a_L$ and P_P < ß, then d = ß - P_P, and a small increase in P_N increases profits without reducing d. Hence, $a_H > P_N \geq a_L$ and P_P < ß is not a profit-maximizing strategy. Conversely, if $P_N < a_L$ and P_P < ß, d = ß - P_P + (a_L - P_N)(M+1), and an increase of e in P_P accompanied by a decrease of e/(M+1) in P_N increases profits without changing d. Hence, having both P_P < ß and $P_N < a_H$ is never a profit-maximizing strategy. €

The intuition behind this result is that a retailer has a *sale* in order to attract non-store loyal consumers. Non-store-loyal consumers are attracted to a store by the consumer surplus (d) offered there. The cost to a retailer of offering any d is the profit it could have earned selling both products to store-loyal consumers at their reservation values. For any given d, a retailer will always use the method of providing that d to non-store-loyal consumers that minimizes this cost. It never pays to offer a sale price on the non-perishable higher than a_L, since P_N less than a_H but greater than a_L reduces the profits from selling to store-loyals without offering any consumer surplus to non-store-loyals. When P_N is less than a_L, offering P_P < ß cannot be profitable, since simultaneously increasing P_P by e and reducing P_N by e/(M+1) leaves consumer surplus to non-store loyals unchanged, but increases profits from loyal customers.

In addition to providing a result useful for deriving the pricing equilibrium, Proposition 1 has an empirical implication: Price movements for the perishable and the non-perishable goods should be negatively correlated. Specifically, combining Proposition 1 with the result from Lemma 4 (below) that one product is always on sale in the symmetric equilibrium, we obtain the result that whenever the non-perishable price changes, the perishable price should move in the opposite direction. The remainder of this section is devoted to explicitly deriving the pricing equilibrium. The next two lemmas provide lower bounds for the pricing distributions for the two goods.[13]

Lemma 1: The lowest price the retailer would ever charge for the non-perishable is $\underline{P}_N$, where

$$\underline{P}_N = \frac{?a_H - J(1-?)ß}{? + J(1-?)(M+1)}.$$

Proof: The sale price must yield profits at least as great as the profits from not having a sale. Note that Proposition 1 implies that if $P_N < a_L$, then P_P = ß, and the lowest price that retailer j will ever find it profitable to charge for the non-perishable sets

[13] The analysis here does not require P_N be positive. Since we interpret P_N as the margin on the non-perishable, $P_N < 0$ does not imply a negative price. Moreover, given our assumption that non-loyals buy both goods at the same store if (ß - P_P) $\geq$ 0 and (a_L - P_N) > 0, P_N less than zero, but greater than $\frac{-P_P(1-?+?/J)}{(1-?)(1+M)+?/J}$ might be profitable. Hence, using the non-perishable as a "loss leader" may be profitable.

$$\frac{?}{J}(a_H\%\text{ß}) \;\; ' \;\; \frac{?}{J}(P_N\%\text{ß}) \;\%\; (1\&?)[(M\%1)P_N\%\text{ß}]$$

where the left-hand side is the profits from not having a sale, and the right-hand side is the profits from having a sale if it were certain that the retailer would be offering the higher d. Simplification yields

$$\underline{P}_N \;\; ' \;\; \frac{?a_H\&J(1\&?)\text{ß}}{?\%J(1\&?)(M\%1)} \text{ €}$$

It follows from Lemma 1 that a necessary condition for retailer j to place the non-perishable on sale is that

$$a_L \geq \frac{?a_H\&J(1\&?)\text{ß}}{?\%J(1\&?)(M\%1)}. \qquad (2)$$

Since the right-hand side of equation (2) is decreasing in M, a sale on the non-perishable becomes profitable for a larger range of parameter values as M rises. The intuition is that as M grows, the ratio of the quantity of the non-perishable bought by loyals to the quantity bought by non-loyals (who only buy the non-perishable during sales) at $P_N < a_L$ falls. That is, the profit from selling to new customers increases with M, while the lost profit from not charging a_H to loyal customers is independent of M.

Lemma 2: The lowest price any retailer will ever charge for the perishable is $\underline{P}_P \;\; ' \;\; \text{ß}\frac{?}{?\%J(1\&?)}$.

Proof: Following the same logic as Lemma 1, the lowest P_P that retailer j will ever choose sets

$$\frac{?}{J}(a_H\%\text{ß}) \;\; ' \;\; \frac{?}{J}(P_P\%a_H) \;\%\; (1\&?)P_P$$

Solving for $\underline{P}_P$ yields $\underline{P}_P \; ' \; \text{ß}\frac{?}{?\%J(1\&?)}$. €

Proposition 1 along with Lemmas 1 and 2 implies that the maximum d any retailer will offer is max{(M+1)(a_L - $\underline{P}_N$), ß - $\underline{P}_P$ }. Let d($\underline{P}_N$) = max{0,(M+1)(a_L - $\underline{P}_N$)} and d($\underline{P}_P$) = ß - $\underline{P}_P$, be the maximal consumer surplus available through *sales* on the two products. Note that d($\underline{P}_P$) is necessarily positive. As shown below in Lemma 4, there is no pure strategy equilibrium in pricing. Rather, in equilibrium, each retailer either chooses a $P_N < a_L$ or a $P_P <$ ß in order to attract non-loyals. Since non-loyals choose between stores on the basis of ds, retailer competition can be characterized in terms of these ds. In addition, for any given d, the retailer will choose the combination of P_N and P_P that maximizes his profits. The next lemma deals with the properties of the profit-maximizing prices for any d. Define L(d) as the pair of prices (P_P, P_N) that lead to maximal retailer profit for any given d, as long as that profit yields profits above the retailer's reservation profit (i.e., hence, L(d) is only defined for $P_P \geq \underline{P}_P$, and $P_N \geq \underline{P}_N$). Lemma 3 defines the properties of L(d) :

Lemma 3: Let $\bar{d} / \frac{(M\%1)}{M}(a_H \& a_L) \& \frac{(M\%1)^2}{M} \frac{J(1\&?)}{?} \; Pr(\bar{d}) \; a_L$, where $Pr(\bar{d})$ is the probability that retailer j attracts non-loyals when it offers $d_j = \bar{d}$. Then

a. $\bar{d} \geq 0$,

b. If $d \leq \bar{d}$ then L(d) = {ß - d, a_H} for all d < d($\underline{P}_P$),

c. If d($\underline{P}_N$) > $\bar{d}$ then L(d) = {ß, a_L - d/(M+1)} for d such that d($\underline{P}_N$) > d > $\bar{d}$.

Proof: see appendix.

Corollary: In the symmetric equilibrium,

a. $\bar{d}$ = (M+1)/(?M)[?(a_H - a_L) - a_L J(1-?)(M+1)(G($\bar{d}$))$^{J-1}$] > 0.

b. Regardless of ß, L(d) = {ß - d, a_H) for d sufficiently small.

Proof: See appendix

Lemma 3 and its corollary show that in the symmetric equilibrium there is always a range of d for which putting the perishable on sale is the most profitable means of offering that d to non-loyal customers. In addition, parts b and c of Lemma 3 state that if the consumer surplus associated with the lowest price the retailer would ever charge for the non-perishable is sufficiently large (greater than $\bar{d}$), then the retailer will offer the non-perishable on sale when it offers a large amount of consumer surplus to non-loyals (d > $\bar{d}$) and place the perishable on sale for d< $\bar{d}$. The logic is that it is costly (in terms of foregone profit from loyals) to offer any consumer surplus to non-loyals by setting P_N below a_L, since the retailer has to sacrifice at least $?(a_H - a_L)/J$. In contrast, offering small d by setting P_P below ß entails only a small reduction in profits from loyals. However, once $P_N < a_L$ the cost in terms of foregone profit from loyals of offering additional d is small. $\bar{d}$ defines where these two factors just offset one another.

The results from Lemma 3 and its corollary allow us to write d in terms of P_N and P_P. Specifically, $d = ß - P_P$ for $d < \bar{d}$, and $d = (M+1)(a_L - P_N)$ for $d > \bar{d}$. Creating this mapping between prices and d allows us to use the following result (due to Varian 1980)).

Lemma 4: There is no point masses in the symmetric equilibrium strategy for d.

Proof: See Varian (1980), Proposition 3.

The intuition behind Lemma 4 is best seen by assuming the contrary; that there is specific $\tilde{d}$ which is offered with a finite probability. In that case, if a deviant store offered a slightly higher d, say $\tilde{d}$ % e, with that same probability it would get the business of all of the non-loyals customers when all of its rivals tied (which occurs with finite probability), while the loss due to the price reduction necessary to obtain the higher d is arbitrarily small. Lemma 4 says that in the symmetric equilibrium, retailers do not offer any specific d with a finite probability; instead in every period d is drawn from a common continuous distribution function, G(d).[14] This implies that d = 0 cannot be offered with a finite probability; that is, one product is always on sale. Further, Lemma 3 implies that G(d) can be decomposed into two cumulative distribution functions; $G(d) = 1 - F_1(P_N)$ for d\$$\bar{d}$ and $G(d) = (1-?)(1 - F_2(P_P))$ for $d<\bar{d}$. Proposition 2 derives these two distribution functions.

Proposition 2. Let $F_1(P_N)$ be the distribution of non-perishable prices and $F_2(P_P)$ be the distribution of perishable prices in the symmetric equilibrium.

a. If $a_L > \underline{P}_N$ and $d(\underline{P}_N) > \bar{d}$, then retailer j puts the non-perishable on sale with probability ?,

[14] Note that G(d) changes over time with changes in M, as detailed below.

$$? = 1 - \left[\frac{\frac{?}{J}(a_H - P_N(\bar{d}))}{(1-?)[(M+1)P_N(\bar{d}) + \beta]} \right]^{\frac{1}{J-1}}$$

where $P_N(\bar{d}) = a_L - \bar{d}/(M+1)$. When the non-perishable is on sale, $P_P = \beta$.

b. If $a_L > \underline{P}_N$ and $d(\underline{P}_N) > \bar{d}$, then the cumulative distribution function for P_N is

$$F_1(P_N) = \begin{bmatrix} 1 - \left[\frac{?(a_H - P_N)}{J(1-?)[(M+1)P_N + \beta]} \right]^{\frac{1}{J-1}} & \text{for } P_N \in [\underline{P}_N, a_L - \frac{\bar{d}}{M+1}], \\ ? & \text{for } P_N \in (a_L - \frac{\bar{d}}{M+1}, a_H), \\ 1 & \text{for } P_N \in a_H \end{bmatrix}$$

c. If either $a_L < \underline{P}_N$ or $d(\underline{P}_N) < \bar{d}$, then $? = 0$ and $F_1(P_N) = 0$ for $P_N < a_H$ and $F_1(a_H) = 1$.

d. With probability 1-? retailer j sets $P_N = a_H$, and chooses P_P according to the distribution

function $F_2(P_P) = 1 - \left[\frac{d?}{J(1-?)P_P} \right]^{\frac{1}{J-1}} (1-?)^{-1}$.

Proof: a and b. From Lemma 4, we know that d is randomly drawn from a continuous distribution with support $(0, \max\{\beta - \underline{P}_P, (M+1)(a_H - \underline{P}_N)\})$. In equilibrium, the profits from charging each price for which the density function is positive must be equal to the profits from charging $P_N = a_H$ and $P_P = \beta$, which are equal to $?[\beta + a_H]/J$. To calculate G(d), note that by Proposition 1, retailer j will put at most one good on sale. If $a_L > \underline{P}_N$ and $d(\underline{P}_N) > \bar{d}$, then retailer j will sometimes put the non-perishable on sale. Specifically, Lemma 3 implies that whether P_N or P_P will be lowered in order to generate consumer surplus of d depends on d. For $d > \bar{d}$, d is obtained by setting $P_N < a_L$. Given this result, when retailer j chooses a $d > \bar{d}$, the probability that a rival offers more consumer surplus is equivalent to the probability the rival offers a lower P_N. Hence for $d > \bar{d}$, $G(d) = 1 - F_1(P_N)$, where $F_1(P_N)$ is the common c.d.f. for P_N. To determine $F_1(P_N)$, note that any P_N for which the density function is positive must yield the same profits as can be obtained by not holding a sale. Hence, the distribution function for P_N, conditional on a sale occurring on the non-perishable must solve

$$\frac{?}{J}(a_H+ß) = \frac{?}{J}(P_N+ß)+(1-?)[(M+1)P_N+ß](1-F_1(P_N))^{J-1}$$

Solving for $F_1(P_N)$ yields

$$F_1(P_N) = 1-\left[\frac{\frac{?}{J}(a_H-P_N)}{(1-?)[(M+1)P_N+ß]}\right]^{\frac{1}{J-1}}$$

The lower bound for the support is the lowest price that it will ever be profitable to charge. As Lemma 2 shows, this price is

$$\underline{P}_N = \frac{?a_H-J(1-?)ß}{?+J(1-?)(M+1)}$$

The highest P_N for which $G(d) = 1 - F_1(P_N)$ corresponds to the d for which it is equally profitable to have a sale on the perishable and non-perishable, or $P_N = a_L - \bar{d}/(M+1)$. For any $d < \bar{d}$, it will be more profitable to lower P_P rather than P_N, so that letting $? \equiv F_1\left(a_L-\frac{\bar{d}}{M+1}\right)$, we know that $F_1(P_N) = ?$ on the open interval $\left(a_L-\frac{\bar{d}}{M+1}, a_H\right)$, and $F_1(a_H) = 1$. By Proposition 1, when $P_N < a_L$, $P_P = ß$.

c. If $a_L < \underline{P}_N$ then it is not profitable to put the non-perishable on sale, and if $d(\underline{P}_N) < \bar{d}$ it is more profitable to put the perishable on sale than the non-perishable. In either case, retailer j will not put the non-perishable on sales, instead charging $P_N = a_H$.

d. Because G(d) is a continuous function, Proposition 1 implies that there is no point mass at d = 0, and that the perishable must be on sale whenever $P_N = a_H$. To solve for $F_2(P_P)$, the c.d.f. of P_P, first note that expected profits when the perishable is on sale at $P_P = ß - d$ are $?(ß - d + a_H)/J + (1-?)G(d)^{J-1}(ß - d)$. In equilibrium, this must equal the expected profits from not having a sale so that

$$G(d) = \left[\frac{d?}{J(1-?)(ß-d)}\right]^{\frac{1}{J-1}} \tag{3}$$

To relate $F_2(P_P)$ to G(d), note that if retailer j puts the perishable on sale, a rival might offer more consumer surplus either by putting the non-perishable on sale, or by offering a lower perishable price. This means that the probability that any one rival offers more consumer surplus than retailer j is $1 - G(d) = ? + (1 - ?)(F_2(P_P)) \Rightarrow G(d) = (1 - ?)(1 - F_2(P_P))$. Using (3) this implies

$$F_2(P_P) \,'\, 1\&\left[\frac{(ß\&P_P)?}{J(1\&?)P_P}\right]^{\frac{1}{J\&1}}(1\&?)^{\&1}.€$$

Proposition 2 shows that, just as in the Conlisk et al., Sobel and Pesendorfer models, the profitability of alternative prices for the non-perishable depends on M, the length of time since the previous sale by any retailer on the non-perishable. In the period immediately following a sale on the non-perishable, it may be unprofitable for any retailer to put the perishable on sale; so that prices for the perishable will vary across retailers. This pattern will persist for several periods, but it eventually becomes profitable for one or more retailers to put the non-perishable on sale. The probability and depth of the sale is increasing in the time since any retailer put the non-perishable on sale. As long as $a_L > \underline{P}_N$, the probability of a sale and the cumulative distribution function for any $P_N < a_L$ is strictly increasing in M. Additionally, Lemma 1 implies that the lower bound for the support of the distribution of P_N declines as M rises. Example 1 presents an illustration of the equilibrium, and how the price distributions change over time.

Example 1: Suppose that $a_H = 5$, $a_L = 2$, ß = 1.5, ? = .75 and J = 2. This implies that the perishable price will be at least .9 (i.e., $\underline{P}_P = .9$), while the lower bound on the support for the non-perishable price distribution ($\underline{P}_N$) depends on M. At M = 1, it turns out that the non-perishable will not be discounted, because $\bar{d}$, the level of consumer surplus that makes sales on the two products equally profitable, is greater than the consumer surplus associated with the maximum profitable discount for the non-perishable. Specifically, $\bar{d} = 0.604$, which means that the P_N which generates $\bar{d}$ is 1.697. This is below the lowest price a retailer could ever profitably charge for the non-perishable when M= 1 (for M = 1, $\underline{P}_N = 1.71$). Hence, for M = 1, d takes on a value between 0 and 0.6, and d is always created by lowering P_P.

As the length of time since the last sale on the non-perishable increases, the profitability of putting the non-perishable on sale rises. For example, when M = 2, $\underline{P}_N = 1.33$, and a sale on the non-perishable would be profitable if $\bar{d} < (M+1)(a_L - \underline{P}_N) = 2 * .67 = 1.34$. As shown in Table 1, for M = 2 $\bar{d} = 0.46$, so that this inequality is satisfied. That is, for d between 0.46 and 1.34, d is created by lowering P_N, and for d between 0 and 0.46, d is created by lowering P_P. The probability of a sale on the non-perishable (?) is .327 when M = 2. If there is no sale on the non-perishable when M = 2, then since $\bar{d}$ is decreasing in M, and (M+1) (a_L - $\underline{P}_N$) is increasing in M, $\bar{d}$ will be less than (M+1) (a_L - $\underline{P}_N$) for all M > 2. In fact, the probability of holding a sale on the non-perishable is nearly 50% for M=3, almost 59% for M=4, and about 65% for M=5.

Table 1

M	$\underline{P}_N$	$\bar{d}$	?	P_N ($\bar{d}$)
2	1.333	.464	.327	1.845
3	1.091	.38	.491	1.905
4	.923	.323	.589	1.935
5	.8	.281	.654	1.953

Figure 1 portrays the c.d.f. for these four values of M. As the corollary to Lemma 3 implies, ? is strictly less than 1, so that for any M, there is a positive probability that the perishable will go on sale. The c.d.f. for the perishable price changes with M though its effect on ?. The c.d.f. for P_P for values of M between 1 and 5 is portrayed in figure 2. €

The pattern of prices illustrated in this example suggests several tests of the model. First, the probability of a sale on a non-perishable should be an increasing function of the elapsed time since the previous sale on that good by any retailer. Second, F_1 implies that the probability and depth of a sale of the non-perishable is increasing in J. Similarly, F_2 implies that the depth of sale on the perishable is increasing in J. Hence, other things equal, a more concentrated market will have fewer and shallower sales. Third, a price reduction on a perishable will be less likely in a period in which there is a price reduction on the non-perishable. Finally, Proposition 2 implies that a non-perishable is more likely to have the same price in consecutive periods than a perishable, and that conditional on a price reduction occurring, the average reduction will be larger for the non-perishable. We examine these final two predictions in the following section.

Figure 1

$F_1(Pn)$ Values for Alternative M

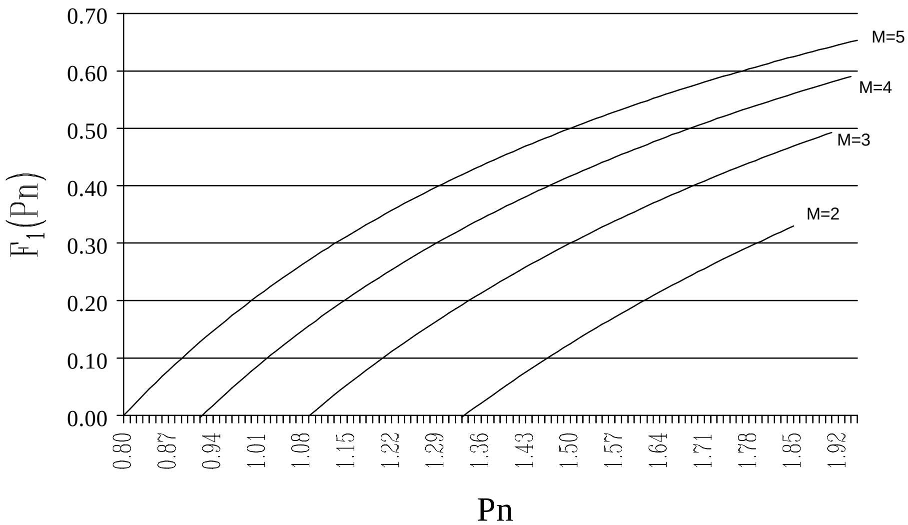
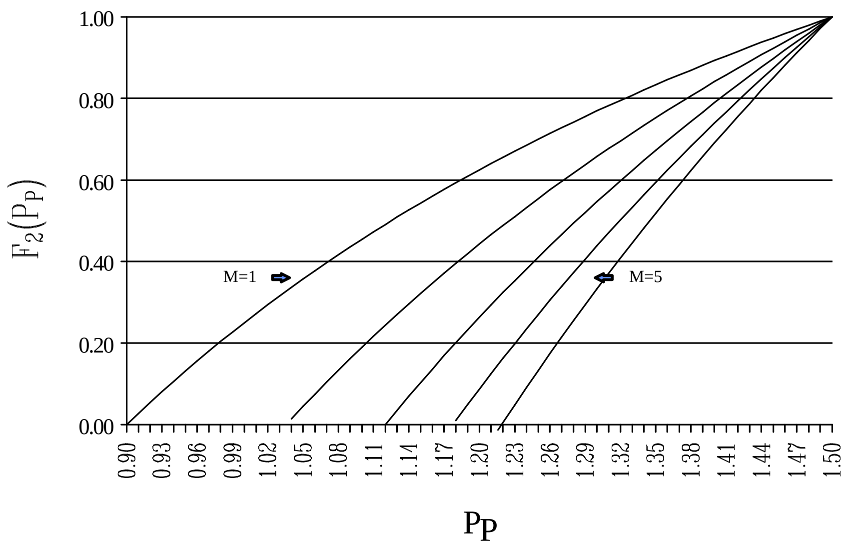


Figure 2

$F_2(P_P)$ Values for Alternative M



IV. Empirical Evidence

Section III develops a model showing how retailers compete for customers through the pricing of both perishable and non-perishable goods. In this section, we present some evidence relating to the model's predictions. An obvious difference between the model and actual food retailers is that a typical supermarket sells thousands of individual items. This implies that the empirical counterparts to the perishable and non-perishable goods of the model are likely to be bundles of goods. For example, because of consumer heterogeneity (e.g., non-loyals may differ in their preferences for beef vs. fish) or differences in "perishability" (e.g., between butter and bananas), more than one perishable may be on sale at any moment in time in order to attract "non-loyal" customers with different tastes. This implies that the price of any product, or group of products will not correspond exactly to the predictions of the model. Nevertheless, the basic intuition of why sales on non-perishables are innately different from sales on perishables developed in the model does yield predictions about price movements for these two classes of goods.

The data we use comes from a public use data set provided by A.C. Nielsen.[15] This data set contains daily prices and "category shares" for several categories of goods at the individual store level for two medium-size Midwestern cities (Springfield, MO and Sioux Falls, SD). Ideally, the perishable products would have very short shelf lives, e.g. lettuce, whereas the non-perishables could be stored for a long period of time without deteriorating, e.g. peanut butter. There were many products in the data set that appear to be good candidates as non-perishables,[16] however, there was only one product (margarine) that met our definition of a perishable. While margarine can be stored for a considerable length of time, it is still perishable in that it must be refrigerated to be stored. Because it must be refrigerated, it is more costly to store than truly non-perishable products. Consequently we use margarine as the perishable grocery product. We have chosen peanut butter as the non-perishable product for several reasons: Peanut butter and margarine have similar price points, both margarine and peanut butter have a number of brands with significant value to

[15] The data can be found at the ftp site: gsbper.uchicago.edu.

[16] These included peanut butter, ketchup, canned tuna, sugar and facial tissue.

consumers,[17] and both have similar weekly average consumption. Within these categories, we focused on the three branded product/sizes with the largest market shares. In both of these cities, there were multiple supermarket chains. Prices within each chain were very highly correlated, and consequently each store within each chain cannot be considered an independent observation. For this reason, we construct one price series for each chain. There are 5 chains in Sioux Falls, and 4 chains in Springfield. The data set covers the 124 week period from January 23, 1985 through June 3, 1987. Table 2 presents descriptive statistics from the data set.

Using these data, we examine several implications of the model. Proposition 2 predicts that perishable goods will go on sale more frequently, but at smaller discounts, than non-perishable goods. Figure 3 presents the pattern of prices for Parkay margarine in Springfield, while figure 4 presents it for Peter Pan peanut butter. The pattern of prices for these two products seem to fit the predictions of the model. Prices for Parkay tends to oscillate from week to week over a relatively small range. In contrast, Peter Pan prices tend to be constant for long periods of time, followed by brief but significant price reductions (typically lasting one or two weeks).[18] One other pattern can be observed from examining figures 3 and 4; sales do not appear to be correlated across stores. That is, it is rare for two stores in a city to lower Peter Pan price in the same week. To the extent wholesale prices are common to all retailers in a market, this supports the premise, central to testing the model, that retail price changes are largely driven by changes in retail margins, rather than changes in wholesale prices.

We formally test the prediction that sales on the non-perishable will be less frequent, but more substantial than sales on the perishable in two ways. First, we examine the number of weeks in which the price was exactly the same as the previous week's price. Looking at the top three brands, the probability of "no change" in price is significantly higher for peanut butter than stick margarine, as shown in table 3. For example, in Springfield, the price of peanut butter remained unchanged in 88.8% of weeks, while the comparable number for stick margarine is 77.7%. This

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[17] There are only two significant brands of ketchup, and one brand of sugar.

[18] Pesendorfer (1997) finds a similar pricing pattern for ketchup.

difference is significant at the 1% level.[19] In table 4, we see this result remains if "no change" in prices is redefined to be *none* of the top three brands of peanut butter or stick margarine in a chain experience a change in price from the previous week. For example, in Springfield in 71.4% of the weeks none of the top three brands of peanut butter changes price, while in 51.1% of the weeks none of the top three brands of margarine changes price. Again, the difference is significant at the 1% level.

The second way we operationalize this prediction is to examine the average price change, conditional on a non-zero change in price. The appropriate measure of discount is the absolute price reduction (rather than percentage) since non-loyal consumers choose among supermarket based on the absolute comparison of total expenditures. As table 5 shows, conditional on there being a price reduction, peanut butter prices were 26.01 cents lower than the previous week in Springfield, while margarine prices were only 15.31 cents lower. This difference is significant at the 1% level.

Two recent papers analyze retail price movements for groups of retail products. While the frequency and magnitude of price changes for different kinds of products is not the focus of their work, their findings have relevance to the questions of interest here. Warner and Barsky (1995) analyze price movements for 7 infrequently-purchased durable goods (such as televisions, drills, and cameras). They find that most price reductions are short-lived, fairly large (between 8 and 25 percent) and followed by a return to pre-sale prices - a pattern broadly consistent with Conlisk et al. (1984), Sobel (1984), Pesendorfer (1997), and the model present here. Lach and Tsiddon (1996) analyze retail prices for meat (a perishable) and liquor (a non-perishable) sampled at monthly intervals for a group of specialty stores in Israel for 1978/79. The difference in time periods and countries between their data and ours requires two caveats be recognized before making any comparisons. First, information technology had progressed significantly between their sample period and ours, so that a retailer's cost of a making a price change was undoubtably much lower during our sample period. Perhaps because of these menu costs, Lach and Tsiddon find that on average, meat retailers adjusted their prices only every other month, and liquor prices were adjusted only every 4-5 months. Second, in contrast to our data in which overall prices were stable, overall inflation in

[19] The results of all of the hypothesis tests appear in table 5.

Israel was nearly 4% per month for the period examined by Lach and Tsiddon.[20] The consequence of these two facts is that a nominal price decrease would constitute a large decline in real prices (especially for liquor, due to the less-frequent price changes), and not surprisingly, the overwhelming proportion of the price changes in their data were in the upward direction. Interestingly, Lach and Tsiddon find that real price reductions large enough to cause nominal prices to fall are significantly more likely for liquor than meat, even though the lower frequency of price changes for liquor would imply the reverse. We view this as evidence that, as in our data, the probability of a large price reduction is higher for non-perishables than perishables.

The third implication we test is that price changes for the two products are negatively correlated. Proposition 1 along with Lemma 4 implies that exactly one good will be sold below the store-loyal's reservation price (more generally, below the profit-maximizing price to store-loyals) during each period. Hence, in any period in which the price of the non-perishable is lower than in the previous period, the perishable's price should be higher than in the previous period. Conversely when the non-perishable price is higher than in the previous period, the perishable price should be lower. The reverse relationship is similar but noisier; the theory implies that there are many different *sale* prices of the perishable but the price of the non-perishable is the same whenever the perishable is on sale. Hence, a higher perishable price may be associated with a lower non-perishable price, or an unchanged non-perishable price.

Supermarkets sell a large number of perishable and non-perishable items, and in any given week typically offer more than one hundred items at sale prices. As noted above, if non-loyal consumers are heterogenous, putting certain combinations of perishables and non-perishables on sale may be a profitable means of attracting different groups of non-loyals. For this reason, the relationship between the prices of a specific perishable and a specific non-perishable may be noisier than suggested by the theory. In addition, the fact that margarine is less than an ideal perishable product makes testing the theory more difficult. Subject to these issues, we examine the relationship between margarine and peanut butter price changes using two similar measures. The first involves estimating a simple regression model. Specifically, we create indicator variables equal to -1 or 1 if

[20] Peanut butter prices increased about 5% per year, and margarine prices declined by 5% per year in our sample.

the price of any brand of a product decreased or increased, respectively, in a given time period at a given store, and zero if all prices remained the same. We then regress the indicator variable for margarine on the indicator variable for peanut butter.[21] Two versions of the model are estimated; one using all of the observations (models 1 and 2) and one using only those observations where peanut butter price changes (models 3 and 4). The results appear in table 6. As the theory predicts, the coefficient on the peanut butter indicator is negative, and is significant at the .05 level in models 1 and 2 and .1 level in models 3 and 4. The coefficients are about .08 in all 4 cases, which suggests that margarine prices are 4 percent more likely to rise (or fall) in weeks in which peanut butter price fell (or rose) than in other weeks. However, given the weak explanatory power of the model (the R-squared is less than .02 in all specifications), we view these results as providing only moderate support for the theory developed here.

Our second method of examine this prediction is to calculate the mean price change of one product given a price increase (or decrease) in the other. As noted above, comparing the mean change in margarine prices conditional on a change in peanut butter prices is a better test than the reverse, because while all peanut butter price changes will be associated with margarine price changes, not every margarine price change will be associated with a peanut butter price change. The conditional means are presented in table 7. The comparisons of margarine prices conditional on peanut butter price changes are in the direction predicted by the theory, but only the effect associated with peanut butter price increases is statistically significant. As expected, the comparison of peanut butter price changes conditional on a margarine price change is less clear. Margarine price decreases are associated with statistically significantly higher peanut butter prices, consistent with the theory. However, conditional on a lower margarine price, peanut butter prices are slightly lower, contrary to the theory (although the effect is not statistically significant). Again, these results provide some modest support for the theory.

Finally, we examined the empirical validity of the assumption that price changes represent changes in retail margins, rather than changes in wholesale price. We test this by looking at the correlation of price changes across stores for a given product. Under the assumptions that (1) prices

[21] We also include an indicator for stores in Springfield, Missouri, however, it is never significant and its inclusion does not affect the coefficient on the peanut butter indicator.

to retailers (wholesale price) move together in each city, and (2) wholesale price changes are reflected in retail prices changes with a lag that is common across all retailers, we would expect to see retail price changes that are highly correlated if sales were primarily driven by wholesale price changes.[22] As figures 3 and 4 suggests, retail price changes are not highly correlated. Tables 8a-8d show the correlations of price changes across stores for the six products in each of the two markets. Nearly half of the correlations are negative, and only 3 of the 96 are greater than .25. This suggests that retail price changes were not primarily driven by changes in wholesale prices.

While we have not been able to test all of the model's implications, the tests we did perform lend empirical support for several of the model's key predictions. First, in every case, the perishable product (margarine) goes on sale more often than the non-perishable (peanut butter). Second, when there is a price change, the absolute value of the average price change is always larger for the non-perishable than the perishable. Further, the evidence suggests that peanut butter and margarine prices changes are negatively associated. In addition, because the contemporaneous correlations of price changes of individual items across stores were typically small (and sometimes negative), it appears safe to conclude that most of the grocery store price changes we observe for peanut butter and margarine are the result of retailers changing margins, rather than manufacturers changing wholesale prices. Together, this evidence indicates sale behavior is an important aspect of retail competition and likely the greatest cause of the observed variation in retail prices. Further, it appears to be the case that retailers pursue different pricing strategies for different types of grocery items, likely related to product characteristics.

V. Conclusion

This paper models price competition among multi-product retailers. We find the intuition used to explain *sales* in models with only one product generalizes to the multi-product setting.

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[22] The assumption that in each city all retailers' wholesale prices move together is based on our understanding of industry practices, along with legal restrictions on differential pricing due to the Robinson-Patman Act. Finally, to the extent that the assumption is incorrect, it would suggest that manufacturers, rather than retailers, were attempting to exploit differences among consumers. Such behavior by manufacturers would be similar to the behavior of retailers in our model.

Examining the multi-product environment is useful because retailers clearly use different pricing policies for different products. We show that such differences can be explained by product characteristics, and also that there is a relationship between the prices charged for different products at any point in time. As such, the model yields a richer set of implications than does the single product case. For example, the theory predicts a negative correlation between perishable and non-perishable prices at a specific supermarket.

We examined this and several other implications using publicly-available pricing data. Consistent with the theory, prices for the non-perishable (peanut butter) and the perishable good (margarine) seem to be negatively correlated. In addition, as the theory implies, price changes are significantly less frequent for the non-perishable, but when changes do occur, they are larger in magnitude for the non-perishable.

We view this evidence as suggesting that price discrimination by intertemporal price changes is one function served by sales in the food retailing industry. In addition to providing us with an understanding of how retailers compete, this view of retail competition has several important policy implications. For example, to the extent our findings are confirmed in future work, it would have several implications for merger analysis. One relates to the correct interpretation of demand elasticities derived from scanner data. Estimates of brand-specific elasticities and cross-elasticities have become a common component of the Federal Trade Commission and U.S. Department of Justice merger review process. Evidence regarding these elasticities is often presented to the agencies by representatives of the merging parties or an interested third party in order to demonstrate the likely consequences of combining two competing brands of a product (e.g. canned soup) under common ownership. The data used in such estimation is typically weekly scanner data on transaction prices and quantities. Such elasticities can be thought of as *purchasing* elasticities; the responsiveness of consumer's buying patterns to changes in prices. If inventorying by consumers is important, these elasticities can be quite different from *consumption* elasticities and it is the latter elasticities which are relevant to merger analysis. For example, if the scanner data covers a period in which a retail price moves between a *regular* price and a *sale* price, then the measured elasticity will reflect purchases by individuals who buy at sale prices and then inventory a non-perishable item (e.g. canned soup or peanut butter) for consumption during the non-sale periods. Even if every individual had a completely inelastic consumption demand, such a study might well demonstrate a significant purchasing elasticity. What is relevant for a merger among manufacturers, however, is what would happen if the entire price schedule changed, and this is not measured by the purchasing elasticity.

The second point is relevant for analyzing mergers among retailers. Suppose inter-retailer competition affects the frequency and depth of sales rather than the level of non-sale prices. Then, evaluating the effect of a merger based on comparing prices during narrow pre- and post-merger windows will provide, at best, a noisy measure of the effect of a merger. If many of the items chosen for comparison are those that are infrequently used as sale items, then we may find little or no price effect of the merger. For example, in the Lal and Matutes (1994) model, retailers always charge the monopoly price for items that are never advertised, regardless of market structure. Hence, one would observe no effect of mergers on prices of these goods, regardless of whether the merger reduces retail competition. Moreover, even if all of the items in the sample are those often subject to sales, if one compares prices in a narrow time window following a merger to a similar pre-merger period, one might find significant numbers of both price reductions and price increases, even if the merger reduces competition. In such an environment, a researcher must be careful when constructing the price index used to determine if a merger led to higher prices.

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Appendix

Lemma 3: Let $\bar{d} \equiv \frac{(M+1)}{M}(a_H - a_L) - \frac{(M+1)^2}{M}\frac{J(1-?)}{?}\ Pr(\bar{d})\ a_L$, where $Pr(\bar{d})$ is the probability that retailer j attracts non-loyals when it offers $d_j = \bar{d}$. Then

a. $\bar{d} \geq 0$,

b. If $d \leq \bar{d}$ then $L(d) = \{ß - d, a_H\}$ for all $d < d(\underline{P}_P)$,

c. If $d(\underline{P}_N) > \bar{d}$ then $L(d) = \{ß, a_L - d/(M+1)\}$ for d such that $d(\underline{P}_N) > d > \bar{d}$.

Proof: a. To see that $\bar{d}$ must be non-negative, note that since $a_H > a_L$, $\bar{d} < 0$ would imply that $Pr(\bar{d}) > 0$; i.e., firm j can attract non-loyals by offering negative consumer surplus. This violates consumer rationality.

b. By Proposition 1, retailers will never put both products on sale. The P_P required to generate consumer surplus of d is ß- d. Hence, the profits from putting the perishable on sale to generate d are

$$?_P = \frac{?}{J}(ß + a_H - d) + (1-?)Pr(d_j)(ß - d)$$

Where $Pr(d_j)$ is the probability that retailer j attracts non-loyals when it offers d_j. Since the non-perishable price which yields consumer surplus of d equals $a_L - d/(M+1)$, the profits from putting the non-perishable on sale to generate d are

$$?_N = \frac{?}{J}(ß + a_L - \frac{d}{M+1}) + (1-?)Pr(d_j)(ß + (M+1)a_L - d)$$

Hence, lowering P_P is a more profitable way to generate d if $?_P > ?_N$, or equivalently, if

$d < \bar{d} \equiv \frac{(M+1)}{M}(a_H - a_L) - \frac{(M+1)^2}{M}\frac{J(1-?)}{?}Pr(d_j)a_L$. Finally, note that since $d(\underline{P}_P)$ is the maximal consumer surplus retailer j can profitably offer by setting P_P below ß, L(d) is only defined for d $< d(\underline{P}_P)$.

c. By construction, $?_N > ?_P$ if $d > \bar{d}$. In addition, if $d(\underline{P}_N) > \bar{d}$, then offering a sale on the non-perishable yields higher profits to retailer j than having a sale on neither good, assuming no other retailer offers more than $\bar{d}$ in consumer surplus. €

Corollary: In the symmetric equilibrium,

a. $\bar{d}$ = (M+1)/(?M)[?(a_H - a_L) - a_L J(1-?)(M+1)(G($\bar{d}$))$^{J-1}$] > 0.

b. Regardless of ß, L(d) = {ß - d, a_H) for d sufficiently small.

Proof: a. In the symmetric equilibrium, Pr(d_j) = Prob(d_j < d_{-j}) = (G(d))$^{J-1}$, where G( ) is a distribution function common to all retailers. By Lemma 3.b, $\bar{d}$ is non-negative. Using the definition of $\bar{d}$, $\bar{d}$ = 0 would imply ?(a_H - a_L) = a_L J(1-?)(M+1)(G(0))$^{J-1}$, and hence a mass point at G(0). As Varian (1980) showed, the equilibrium distribution cannot have a mass point (see Lemma 4). Hence, $\bar{d}$ > 0.

b. First note that

$$\frac{?}{J}(a_H\%ß\&d)\%(1\&?)(ß\&d)>\frac{?}{J}(a_H\%ß)$$

for d sufficiently small. If d($\underline{P}_N$) < $\bar{d}$, then either L(d) does not exist or L(d) = (ß - d, a_H). By Lemma 3, L(d) always exists for d sufficiently small, so that L(d) = (ß - d, a_H) for some d. If d($\underline{P}_N$) > $\bar{d}$, then since $\bar{d}$ is greater than zero by (a), we know that $?_N(\bar{d})\,'\,?_p(\bar{d})>0$, and hence that L(d) = (ß - d, a_H) for d less than $\bar{d}$, but greater than 0. €

Table 2: Descriptive Statistics

Variable	Sioux Falls, South Dakota	Springfield, Missouri
Price of Blue Bonnet 4 Pack Stick Margarine	0.569 (0.086) [599]	0.618 (0.088) [495]
Price of Fleischman 4 Pack Stick Margarine	1.15 (0.098) [588]	1.08 (0.077) [495]
Price of Parkay 4 Pack Stick Margarine	0.572 (0.087) [601]	0.589 (0.110) [496]
Price of 18 Ounce Jif Creamy Peanut Butter	1.74 (0.160) [596]	1.84 (0.212) [495]
Price of 18 Ounce Peter Pan Creamy Peanut Butter	1.76 (0.204) [577]	1.78 (0.246) [496]
Price of 18 Ounce Skippy Creamy Peanut Butter	1.68 (0.117) [571]	1.84 (0.228) [495]
Number of Chains	5	4
Number of Weeks	124	124

Standard Deviations is in parentheses, number of observations in brackets, and prices in dollars.

Table 3a: Percentage of Non-Sale Weeks for the Top Three Brands of Peanut Butter and Stick Margarine in Springfield, Missouri and Sioux Falls, South Dakota

Product	Proportion No Price Change in Sioux Falls	Proportion No Price Change in Springfield
Blue Bonnet 4 Pack Stick Margarine	73.7% [593]	78.3% [488]
Fleischman 4 Pack Stick Margarine	84.5% [579]	85.9% [491]
Parkay 4 Pack Stick Margarine	70.9% [595]	68.9% [492]
Average 4 Pack Stick Margarine	76.3% [1767]	77.7% [1471]
Jif 18 Ounce Creamy Peanut Butter	88.6% [588]	92.4% [489]
Peter Pan 18 Ounce Creamy Peanut Butter	81.2% [569]	79.7% [492]
Skippy 18 Ounce Creamy Peanut Butter	86.2% [564]	94.3% [489]
Average 18 Ounce Peanut Butter	85.4% [1721]	88.8% [1470]

Number of observations in brackets.

Table 3b: Percentage of Weeks None of the Top Three Brands of Peanut Butter and Stick Margarine are on Sale in Springfield, Missouri and Sioux Falls, South Dakota

Product	Proportion No Price Change in Sioux Falls	Proportion No Price Change in Springfield
Peanut Butter	67.4% [530]	71.4% [454]
Margarine	51.5% [530]	51.1% [454]

Number of observations in brackets.

Table 4: Comparison of Average Price Increase and Price Decrease for Top Three Brands of Stick Margarine, Tub Margarine, and Peanut Butter for Springfield, Missouri and Sioux Falls, South Dakota

Product	Price Increase Sioux Falls	Price Decrease Sioux Falls	Price Increase Springfield	Price Decrease Springfield
Blue Bonnet 4 Pack Stick Margarine	12.44 (6.75) [75]	-12.00 (6.59) [81]	12.67 (8.15) [52]	-13.16 (8.15) [54]
Fleischman 4 Pack Stick Margarine	15.76 (9.14) [45]	-15.82 (9.64) [45]	7.65 (3.82) [34]	-8.26 (3.42) [35]
Parkay 4 Pack Stick Margarine	11.73 (6.62) [85]	-11.84 (7.15) [88]	19.43 (11.25) [76]	-20.03 (11.88) [77]
Average 4 Pack Stick Margarine	12.87 (7.42) [205]	-12.79 (7.68) [214]	14.79 (10.26) [162]	-15.31 (10.55) [166]
Jif 18 Ounce Creamy Peanut Butter	15.76 (10.19) [38]	-17.79 (12.94) [29]	23.55 (14.94) [20]	-17.12 (11.88) [17]
Peter Pan 18 Ounce Creamy Peanut Butter	19.09 (10.94) [60]	-19.85 (12.43) [47]	32.83 (22.86) [53]	-32.75 (24.89) [47]
Skippy 18 Ounce Creamy Peanut Butter	19.92 (13.19) [40]	-19.79 (12.56) [38]	24.26 (19.76) [15]	-13.31 (10.03) [13]
Average 18 Ounce Peanut Butter	18.42 (11.49) [138]	-19.31 (12.52) [114]	29.26 (21.06) [88]	-26.01 (22.22) [77]

Standard Deviation in parentheses, number of observations in brackets, and price changes in cents.

Table 5: Test Statistics for Comparisons of Margarine and Peanut Butter Sales Behavior in Sioux Falls, South Dakota and Springfield, Missouri.

Test	Mean Margarine	Mean Peanut Butter	T-Statistic
Average Price Increase for Margarine and Peanut Butter are Equal in Sioux Falls	12.87	18.42	5.01
Average Price Decrease for Margarine and Peanut Butter are Equal in Sioux Falls	-12.79	-19.31	5.07
Proportion Of No Price Change Equal for Margarine and Peanut Butter in Sioux Falls	0.763	0.854	6.82
Proportion of No Price Change of any Peanut Butter Equal to Proportion of No Price Change of any Margarine in Sioux Falls	0.515	0.674	5.27
Average Price Increase for Margarine and Peanut Butter are Equal in Springfield	14.79	29.26	6.06
Average Price Decrease for Margarine and Peanut Butter are Equal in Springfield	-15.31	-26.01	4.02
Proportion Of No Price Change Equal for Margarine and Peanut Butter in Springfield	0.777	0.888	8.05
Proportion of No Price Change of any Peanut Butter Equal to Proportion of No Price Change of any Margarine in Springfield	0.511	0.714	6.28

Table 6: Regress Indicator of Change in Margarine Price on
Indicator of Change in Peanut Butter Price

Variable	Model 1	Model 2	Model 3	Model 4
Intercept	0.00766 (0.030)	-0.00106 (0.023)	-0.0371 (0.0557)	-0.0246 (0.0419)
Peanut	-0.0829 (0.0417)	-0.0826 (0.0417)	-0.0793 (0.0418)	-0.0797 (0.0419)
Springfield Indicator	-0.0188 (0.0446)		0.0289 (0.0838)	
R-squared	0.0044	0.0042	0.0127	0.0123
observations	971	971	290	290
Include Observations with no change in Peanut Butter Prices	yes	yes	no	no

Standard Errors are in parentheses. The standard errors are corrected for arbitrary heteroscedasticity (see White (1980)).

Table 7: Change in Margarine or Peanut Butter Price
Conditional on a Peanut Butter or Margarine Price Increase or Decrease

	Mean Change	Observations
Mean Change in Margarine Price Conditional on a Peanut Butter Price Increase	-2.595 (11.914) [-2.781]	163
Mean Change in Margarine Price Conditional on a Peanut Butter Price Decrease	0.464 (12.178) [0.430]	127
Mean Change in Peanut Butter Price Conditional on a Margarine Price Increase	0.284 (17.122) [0.253]	232
Mean Change in Peanut Butter Price Conditional on a Margarine Price Decrease	2.644 (19.152) [2.121]	236

Standard Deviations are in parentheses, t-test that change in price is different from zero is in brackets.
Note that the unconditional mean change for both peanut butter and margarine prices is essentially zero.

Table 8a
Correlations of Peanut Butter Changes Across Chains in Sioux Falls, South Dakota

Brand Name		Chain 1	Chain 2	Chain 3	Chain 4	Chain 5
18 Ounce Jif Creamy Peanut Butter	Chain 1	1.000	-0.115	0.090	0.000	-0.296
	Chain 2	-0.115	1.000	-0.003	-0.204	-0.001
	Chain 3	0.090	-0.003	1.000	0.006	0.225
	Chain 4	0.000	-0.204	0.006	1.000	-0.092
	Chain 5	-0.296	-0.001	0.225	-0.092	1.000
18 Ounce Peter Pan Creamy Peanut Butter	Chain 1	1.000	0.429	0.206	0.021	0.009
	Chain 2	0.429	1.000	0.034	0.045	-0.058
	Chain 3	0.206	0.034	1.000	-0.168	0.090
	Chain 4	0.021	0.045	-0.168	1.000	0.062
	Chain 5	0.009	-0.058	0.090	0.062	1.000
18 Ounce Skippy Creamy Peanut Butter	Chain 1	1.000	0.266	-0.001	-0.416	-0.094
	Chain 2	0.266	1.000	0.031	0.404	0.230
	Chain 3	-0.001	0.031	1.000	0.000	0.000
	Chain 4	-0.416	0.404	0.000	1.000	0.223
	Chain 5	-0.094	0.230	0.000	0.223	1.000

Table 8b
Correlations of Peanut Butter Price Changes Across Chains in Springfield, Missouri

Brand Name		Chain 1	Chain 2	Chain 3	Chain 4
18 Ounce Jif Creamy Peanut Butter	Chain 1	1.000	-0.060	0.135	-0.140
	Chain 2	-0.060	1.000	-0.188	0.207
	Chain 3	0.135	-0.188	1.000	-0.117
	Chain 4	-0.140	0.207	-0.117	1.000
18 Ounce Peter Pan Creamy Peanut Butter	Chain 1	1.000	0.193	-0.024	0.044
	Chain 2	0.193	1.000	0.210	0.160
	Chain 3	-0.024	0.210	1.000	-0.009
	Chain 4	0.044	0.160	-0.009	1.000
18 Ounce Skippy Creamy Peanut Butter	Chain 1	1.000	0.510	-0.008	-0.109
	Chain 2	0.510	1.000	-0.011	-0.004
	Chain 3	-0.008	-0.011	1.000	-0.007
	Chain 4	-0.109	-0.004	-0.007	1.000

Table 8c
Correlations of Margarine Price Changes Across Chains in Sioux Falls, South Dakota

Brand Name		Chain 1	Chain 2	Chain 3	Chain 4	Chain 5
Parkay 4 Pack Stick Margarine	Chain 1	1.000	-0.015	0.054	0.147	0.077
	Chain 2	-0.015	1.000	-0.024	-0.279	-0.098
	Chain 3	0.054	-0.024	1.000	0.057	0.020
	Chain 4	0.147	-0.279	0.057	1.000	-0.050
	Chain 5	0.077	-0.098	0.020	-0.050	1.000
Blue Bonnet 4 Pack Stick Margarine	Chain 1	1.000	0.168	-0.001	0.000	0.045
	Chain 2	0.168	1.000	0.179	-0.010	0.007
	Chain 3	-0.001	0.179	1.000	-0.161	-0.026
	Chain 4	0.000	-0.010	-0.161	1.000	0.194
	Chain 5	0.045	0.007	-0.026	0.194	1.000
Fleischman 4 Pack Stick Margarine	Chain 1	1.000	0.025	0.226	-0.286	0.090
	Chain 2	0.025	1.000	0.000	-0.013	-0.229
	Chain 3	0.226	0.000	1.000	-0.017	0.057
	Chain 4	-0.286	-0.013	-0.117	1.000	0.252
	Chain 5	0.090	-0.229	0.057	0.252	1.000

Table 8d
Correlations of Margarine Price Changes Across Chains in Springfield, Missouri

Brand Name		Chain 1	Chain 2	Chain 3	Chain 4
Parkay 4 Pack Stick Margarine	Chain 1	1.000	0.000	-0.066	0.001
	Chain 2	0.000	1.000	0.056	0.062
	Chain 3	-0.066	0.056	1.000	0.041
	Chain 4	0.001	0.062	0.041	1.000
Blue Bonnet 4 Pack Stick Margarine	Chain 1	1.000	-0.181	-0.052	-0.072
	Chain 2	-0.181	1.000	0.206	0.145
	Chain 3	-0.052	0.206	1.000	0.012
	Chain 4	-0.072	0.145	0.012	1.000
Fleischman 4 Pack Stick Margarine	Chain 1	1.000	0.026	-0.003	-0.106
	Chain 2	0.026	1.000	0.212	-0.075
	Chain 3	-0.003	0.212	1.000	-0.003
	Chain 4	-0.106	-0.075	-0.003	1.000

Figure 3: Time Series of Shelf Prices of Parkay Margarine in Springfield, MO

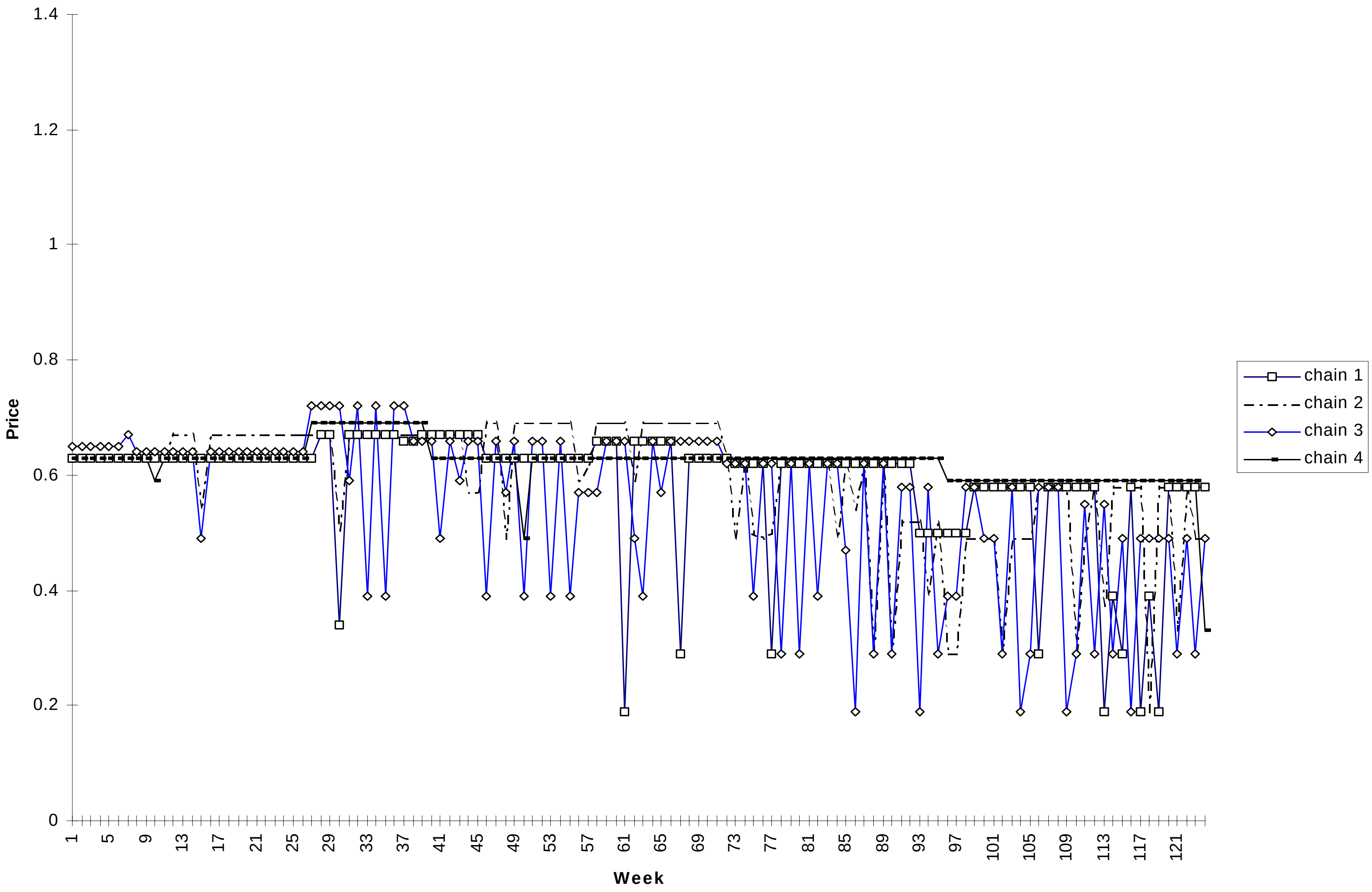

Figure 4: Time Series of Shelf Prices of Peter Pan Peanut Butter in Springfield, MO